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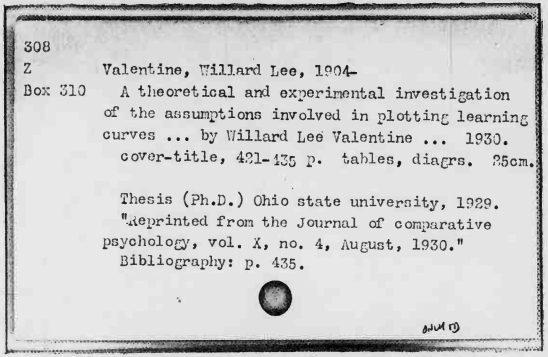
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A Theoretical and Experimental Investigation of the Assumptions Involved in Plotting Learning Curves

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DISSERTATION

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THE GRADUATE SCHOOL OF THE OHIO STATE UNIVERSITY

BY

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A STUDY OF LEARNING CURVES

I. THE APPLICATION OF MEYER'S ARC COTANGENT FUNCTION AND THURSTONE'S HYPERBOLA TO THE MAZE PERFORMANCE OF WHITE RATS[1]

WILLARD L. VALENTINE

Ohio State University

Meyer (1923) has assumed that the arc cotangent function is adequately representative of learning scores. This function may be written

$$t = K \operatorname{arc\,cot} (n + p) + t_l \qquad (1)$$

[where t is the *time* required to perform a unit errorless act, i.e., where all errors have been corrected and the time required to correct them considered in the total, K is a constant, or parameter, n the number of the practice act, p the equivalent previous practice, and t_l the *time* of limiting performance (physiological limit)]. Equation (1) exhibits an inflection point at $(n + p) = 0$, which corresponds to $t = \frac{1}{2}(t_o - t_l)$ where t_o is the time corresponding to $n + p = -\infty$. The existence of the inflection point *in these coördinates*[2] is the moot point in Meyer's equation of the learning function as opposed to Thurstone's.

Thurstone (1919) writes, corresponding to the above, the hyperbola,

$$t = t_l \frac{(n_1 + p) + a}{(n_1 + p)} \qquad (2)$$

---

[1] This work was done at Ohio Wesleyan University under the joint direction of Dr. A. P. Weiss and Dr. F. C. Dockeray.

[2] Thurstone (1919, p. 41) warned against confusing the coördinates. This insidious error has been made by many writers, most recently by Josephine M. Smith,—Which Hand Is the Eye of the Blind? Genetic Psychol. Mono., 1929, v, p. 229.

where the t, t_1 and p are defined as above and $n_1 = (n + 1)$, and a is a constant.[3]

So far as I am aware, no maze studies on white rats have exhibited learning curves containing inflection points. All display the characteristic hyperbolic form. This evidence leads to the formulation of three possibilities:

1. Meyer is in error in assuming the trigonometric equation.
2. All existing data obscure the inflection point through difficulties in technic, either in collecting the raw data, recording it, or displaying it in the form of tables and graphs.[4]
3. The maze problem is not a problem in learning, but only one in improvement.[5] If this were true, most of the learning process would be over before the rat is put in the maze so that just the latter portion of the whole curve would be obtained.[6]

Which of the many possible forms of equations to use to represent any set of data is always a puzzling problem. One arrives at a conclusion, not on the basis of his mathematical knowledge, but on the basis of this knowledge concerning the process itself. If one has reason to believe that learning is a process of substitution, then he is forced to the acceptance of an equation which exhibits an inflection point (S-shaped). This either may or may not be the arc cotangent function, but it must be one that *looks like it*. The available ones at present seem to be the $\Phi(\gamma)$, Pearl's logistic growth curve, Robertson's monomolecular autocatalytic function, and various other exponentials that have no particular designation.

[3] Some liberties have been taken with Thurstone's notation and definition. These are defended in another paper. Suffice it to say here that the n's in Meyer's equation are counted 0, 1, 2, 3,—while in Thurstone's, n_1 is 1, 2, 3, 4. This is entirely a matter of definition and need not confuse.

[4] The latter two are references to the type of thing discussed by Joseph Peterson, Experiments in Ball Tossing, Jour. Exper. Psychol., 1917, ii, 178–224, while the first refers to the arbitrary manner in which errors are counted and for which the time record is adjusted.

[5] This distinction is made on the basis of Meyer's "double stimulation theory," which states that learning consists *not in the improvement* of an already existing function, but rather in the substitution of one function for another. Cf. M. F. Meyer, Psychology of the Other One, 1922, 119; M. F. Meyer and F. O. Eppright. The Equation of the Learning Function, Amer. Jour. Psychol., 1923, xxxiv, 205.

[6] Since this was written, Yoshioka from a different approach has offered a similar interpretation, Jour. Genetic Psychol., 1929, xxxvi, 51–57.

Culler (1928) has concluded in favor of an S-type function, holding that the two apparently contradictory views illustrated above are really complementary. He is, however, speaking in terms of coördinates different from those used in this paper. Others, including Thorndike (1913) and Chapman and Hills (1916), have expressed themselves similarly at least for "complex functions," and McGeoch (1929, 459), on the basis of the recent work of Jete (1928), Goodenough and Brian (1929), Brainard (1927), Wilson (1927), Brown (1928), Bott, Brown & Cohen (1928), concludes . . . . "that at least the following three conditions may operate to produce a positively accelerated learning curve: (A) *very young subjects* (italics mine); (B) subjects of relatively inferior intelligence; and (C) learning problems which are of relatively great difficulty, either intrinsically or because of the method of attack employed. *These three conditions probably reduce to the single common factor that the subject starts the learning with an already possessed (usable) proficiency in the act* which is closer to zero than is usually the case under the conditions yielding negative acceleration [no inflection point]." Thurstone, himself, has recently suggested that the S-shaped curve is more general than the hyperbolic. Assuming that the application of the arc cot function is not in error, then, this paper concerns itself mainly with the possibility that the inflection point has been obscured in previously collected data and incidentally with the third possibility mentioned above.

The youngest rats subjected to rigorous experimentation to date have been 25 days old (Stone's data in Thorndike, 1928). Little or no difference between the learning curves of these and older rats has been demonstrated. If Meyer's hypothesis is true, and if the maze problem is one in learning, then we should be able to make such a demonstration, providing we can get very young rats to run the maze.

According to Meyer, "the probability that p (equation (1)) . . . . is more and more negative corresponds to the greater and greater youthfulness of the animal tested" (Meyer and

Eppright, 1923, 205–6). This is equivalent to saying that the age at which an animal begins a task in formal learning shifts the origin of the curve representing this learning, but has no influence on the form of the curve.

This point was chosen for demonstration because it is critical in the controversy between Thurstone and Meyer. Supposing that we have a learning curve for a 25-day old rat, although, as we shall show, one can fit either an hyperbola or an arc cotangent function to these data, extrapolation to the left produces two entirely different pictures of the genesis of the process. The hyperbola ever rises while the arc cotangent function approaches a finite limit (t_o) as ($n + p$) becomes more and more negative. By taking more youthful animals the experimental points should follow either one or the other of the two curves.[7] One cannot hope, of course, to get reliable results with a single animal. Since litters are more likely to be homogeneous than unrelated animals of the same age, each litter is kept intact in the following experiments. Since variability among litters is reduced to a minimum through long inbreeding, all litters should be fairly homogeneous except with respect to age.

EXPERIMENTAL

A maze of the pattern in figure 1 was constructed as formerly described (Valentine, 1929), and employed in this problem. The pattern was chosen because of its desirable simplicity. It presents 4 paths (providing there is no retracing) from *S–F*. The two middle-length paths are, of course, identical with respect to distance. The time required to traverse from *S–F* was measured.[8] An auxiliary record of the path, retracings, etc., was kept, but is not used here.

[7] But only providing that some structural characteristic of very young animals does not produce a change in the problem. That is to say, a single mathematical function will fit the data only as long as other conditions in the experiment remain relatively constant, n constituting the only systematically varied variable.

[8] Time was kept with a football timer so that if a rat sat down in a corner of the maze, and went to sleep, time out could be taken. This is an effort to control one of many variables in the situation, for it is apparent that with "well motivated" animals such interludes in maze running activity will not occur.

The floor was a piece of 12 x 12 feet B-gauge battleship linoleum, which was coated with clear linoleum varnish. This gave a hard, smooth floor which was washed each day.

The illumination, partly direct, partly indirect, was furnished by 24 equally spaced 60-watt blue lamps arranged 8 feet above the maze floor. The illumination on the maze ranged from 10- to 16-foot candles at different locations as measured by a Macbeth Illuminometer. The illumination gradually dropped from June, 1927, to June, 1928, due to an accumulation of dust, particularly on the white curtain which enclosed the maze within a 7-foot wall.

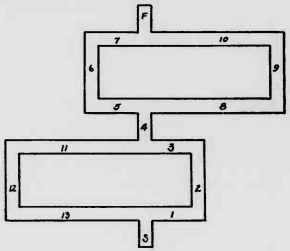

FIG. 1

The illumination inside the curtain, coupled with the fact that there was none outside, permitted an observer to watch the rats through the curtain from the outside, without being in their visual field himself. The floor here was padded with about 3 inches hair-felt which reduced foot noises to a minimum. A platform was also built so that an observer could look down on the maze from above.

THE ANIMALS

The rats, originally of Wistar stock, were all bred in the laboratory. Their ages, ranging from 120 to 18 days, were

accurately known. Each litter was kept in an individual cage. The results of 59 of these animals are reported here. The daily ration consisted of Blackford's Egg Mash mixed with water for the adults and with milk for the pups. Each week they were given greens. Under these conditions they remained sleek, and bred well; both conditions denoting fitness (Evans and Bishop, 1923), even though they remained considerably and consistently under the Donaldson weight norms (Donaldson, 1924).

PROCEDURE

No attempt was made to standardize the feeding in the maze itself. Upon reaching the food box each rat was permitted a few seconds with a variety of food offered there. This "bait" was in the form of table scraps, and as it will be noted, differed from the standard diet which was fed in quantity on a table before the rat was returned to its cage. This departure from the accepted technic was adopted in view of the rather definite preferences for foods exhibited by different rats and the same rats on different occasions. It avoids the loss of weight so often encountered in long experiments, because the rats do not have to be reduced to "utter hunger" in order to obtain consistent performances.

This procedure was varied in the case of the very young rats (18 to 21 days). In these cases the mother remained with the rats over night but was removed in the morning so that they came to the experiment having had neither food nor water during an 8-to-10-hour interval.

The experimental period started about 7 o'clock and ran until 10 or 11 each night. Each rat was given 1 run in each twenty-four hours, and no day was omitted.

In studying p it was impossible to use either a platform escape box or a preliminary maze training since these technics would have influenced the results in an unknown amount. As a consequence there is no reason to expect that the first experimental point falls on, or even near, a curve which fits the terminal data.

RESULTS

The data of table 1 are graphed in figure 2. The litter designated cage 6 contained 7 rats which were 120 days old at the beginning of the experiment. For 17 days the rats were given 1 run a day in the maze. The mean time (t) for all the animals on the first day was 128 seconds (omitted in figure 2) which, at the end of the 17 days, had been reduced to 8 seconds. Here it became

TABLE 1

ARC COT					HYPERBOLA			
n	t	Cal	r	r^2	n_1	Cal	r	r^2
0	128	139.9	9.9	98.0	1	103.0	28.0	784.0
1	72	70.7	1.3	1.7	2	63.3	7.7	59.3
2	50	45.3	4.7	22.1	3	43.6	1.4	2.0
3	47	33.4	13.6	184.9	4	38.6	8.4	70.4
4	28	26.6	1.4	2.0	5	37.6	4.6	21.2
5	25	22.3	2.7	7.3	6	23.4	1.4	2.0
6	27	19.2	7.8	60.8	7	20.5	4.5	20.2
7	26	17.0	9.0	81.0	8	18.3	6.7	44.9
8	16	15.2	0.8	0.6	9	16.3	0.8	0.6
9	14	13.8	0.2	0.0	10	14.0	0.7	0.5
10	20	12.7	6.3	39.7	11	13.9	7.0	49.0
11	13	11.8	1.2	1.4	12	11.0	1.6	2.6
12	12	11.3	0.7	0.5	13	10.2	1.9	3.6
13	10	10.4	0.4	0.2	14	1.5	1.0	1.0
14	9	10.0	1.0	1.0	15	7.0	1.1	1.2
15	8	9.8	1.8	3.2	16	7.2	0.8	0.6
16	8	9.3	1.3	1.7	17	6.0	1.7	2.9
Sum.....				506.1				1066.0

evident that further decrease would be negligible so the experiment with this litter was discontinued.

In each case we are interested in obtaining only enough points so that a curve can be put through them with some degree of reliability. To continue the experiment in every case to 30 or 40 trials is a waste of time, since by experiment, it was determined that these terminal data will always be fitted. Far more critical and differentiating between litters are the first 6 or 8 days.

The protocols show that when the group average was 8 seconds

or lower, the rats were consistently following the shortest path. They also show that after 30 days at 5 trials a day, the minimum individual time is in the neighborhood of 3 seconds by the shortest path, 6 seconds by either of the medium paths, and 9 seconds for the longest.

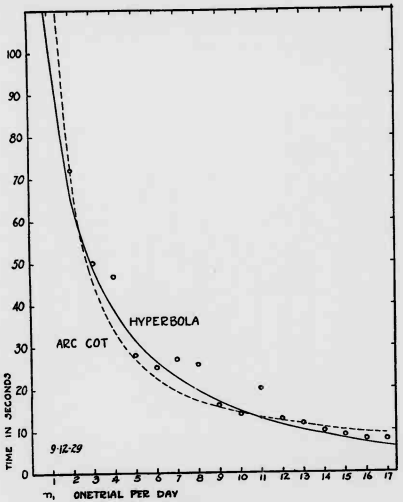

FIG. 2

The column under "Arc cot" headed "Cal" in table 1 indicates the calculated value of the time (t) for each practice period. The method of calculation has already been described (Meyer and Eppright, 1923, 203–222). The values of t correspond to

those given by evaluating the equation $t = 1.8 \cot^{-1}(n + 0.3) + 3.0$ for integral values of n from 0–16. The column head "Cal" under "Hyperbola" gives similar values determined graphically[9] and corresponding to the evaluation of

$$t = -7.6 \frac{n_1 + 1.6 - 36.7}{n_1 + 1.6}$$

The column headed r in each case contains the residual, or the difference between the calculated and experimental value of t. The column r^2 contains the square of these residuals. The sum (Σr^2) serves as an index of comparison between various curves. The P.E. is given by

$$\text{P.E.} = 0.6745 \sqrt{\frac{\Sigma r^2}{n - 0}}$$

where n is the number of observations and 0 is the number of r^2's equal to zero. The P.E. as defined above is the probable value of a residual (r) or the probable difference between an observed and calculated value of t. Whether the precision of either the observations or calculations warrants the use of the above formula for the P.E. is questionable. The concept is introduced here because we will find it useful later in making some important comparisons.

Probably half the people examining figure 2 will be able to suggest a better fit. That is, the curve should be made flatter in the region of $n = 3 - 8$. It is true that smaller residuals thereby would be obtained, but we have considered that further manipulation of the constants to obtain more precise fits is unwarranted in view of the additional time required. The curves presented probably all give residuals within the experimental error. It is considered good practice in curve fitting to get approximately as many experimental or measured points, on one side of the calculated curve as on the other. We have not been unduly concerned with this problem, however, because unlike other experiments, the factors over which we have little or no control all tend to make

[9] The details of this method which need not concern us here will be found in a later paper of this series.

the performance poorer in a learning experiment. If the curve represents the results that would be obtained under ideal conditions, we should expect to measure more points above it than below it.

TABLE 2

ARC COT					HYPERBOLA			
n	t	Cal	r	r^2	n_1	Cal	r	r^2
0	219.0	106.0			1	347.5		
1	66.0	69.4	3.4	11.6	2	57.0	9.0	81.0
2	25.0	35.7	10.7	114.5	3	30.5	5.5	50.2
3	26.0	22.4	3.6	13.0	4	20.6	5.4	29.1
4	17.0	16.2	0.8	0.6	5	16.3	0.7	0.5
5	15.0	13.0	2.0	4.0	6	12.2	2.8	7.8
6	10.0	10.9	0.9	0.8	7	10.0	0.0	0.0
7	8.0	9.4	1.4	2.0	8	8.5	0.5	0.3
8	6.0	8.2	2.2	4.8	9	7.2	1.3	1.7
9	6.0	7.6	1.6	2.6	10	6.4	0.4	0.2
10	6.0	7.0	1.0	1.0	11	5.6	0.4	0.2
11	7.0	6.6	0.4	0.2	12	5.0	2.0	4.0
12	5.0	6.2	1.2	1.4	13	4.5	0.5	0.3
Sum.....				156.5				175.3

TABLE 3

Summary of data

CAGE	NUMBER	AGE	ARC COTANGENT				HYPERBOLA			
			t	K	p	P.E.	t	a	p	P.E.
		days								
1	7	120	3.0	1.8	0.3	3.8	−7.6	−36.7	1.6	5.3
2	7	100	2.0	0.8	−0.8	2.4	−1.2	−58.1	−0.8	2.7
3	7	55	2.0	0.8	−0.5	2.0	−3.7	−24.4	−0.4	2.3
4	7	53	2.0	1.0	0.0	2.7	−1.0	−72.6	−0.4	3.6
5	4	24	3.0	0.7	−0.5	1.8	−2.5	−33.0	0.0	2.5
6	2	24	3.0	0.9	−1.5		−5.5	−10.0	−2.5	
7	4	22	3.0	0.7	−2.0	2.6	+0.2	−483.5	−0.5	
8	3	22	3.0	1.0	−2.0		−18.0	−11.7	−1.0	
9	5	22	3.0	1.0	−2.0		−8.2	−15.6	−2.0	
10	7	20	3.0	0.5	−3.0		−4.0	−23.4	−1.8	
11	4	20	0.0	0.5	−3.0		−2.2	−24.2	−2.6	
12	3	18	3.0	0.7	−4.0		+1.5	+43.4	−2.8	
13	5	18	5.0	0.7	−2.0					

In a similar manner, the t-column in table 2 indicates the mean time for 7 rats, 100 days old. The constants for the arc cotangent equation were determined, and the calculated values of t correspond to an evaluation of

$$t = 0.8 \cot^{-1} (n - 0.8) + 2.0$$

while the hyperbolic values correspond to

$$t = -1.2 \frac{n_1 + 0.8 - 58.1}{n_1 - 0.8}$$

This same analysis, in the original data, is made for each of the 13 litters involved in the experiment. In order to save space and at a sacrifice of effectiveness, we will persent the remainder of the data in tabular form.

Each of the cages as indicated in table 3 is a single litter. The 3 rats in cage 8 were litter mates of the 2 in cage 6; otherwise, the various cages contained unrelated animals.

The P.E.'s are computed for only part of the data because the poor approximation of either function in the other cases makes its value questionable. For those cases in which it has been computed, it is apparent that the arc cotangent function has a slightly higher predictive value than the hyperbola. In every case, the P.E. for the hyperbola is greater.

The values for the parameters in the last cage are not computed because the data do not approximate this function.

For the arc cotangent the value of t_l is in accord with observation, except in one instance. On the other hand, the hyperbola gives values of t_l entirely out of accord with logic. Since it can be shown that t_l in both cases is identical, then this is further evidence that these data are not hyperbolic. If t_l can be interpreted as a lower limit (physiological limit) of the time curve, it is unimaginable that it should be a negative quantity, excepting a small one due to experimental error. The choice of the short maze was thus fortunate in that it clearly exhibits that the hyperbola gives t_l values much too low. In a longer problem this fact would not have been so apparent.

If we choose values of t_l for the hyperbola which are in accord

with observation, then the corresponding hyperbolae do not approximate the data as closely. This would increase the difference between the P.E.'s of the two curves by making those of the hyperbola greater.

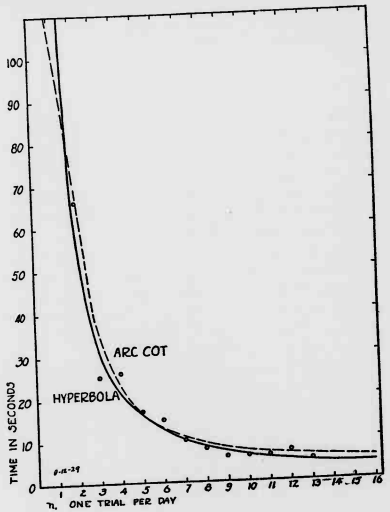

FIG. 3

It is possible to find arc cot functions which give closer approximations by lowering the value of t_1. The data for cage 13, for instance, can be fitted much better if the curve

$$t = 7.0 \cot^{-1} (n + 1.0) - 35.0$$

is used. The trend of the data is much better approximated and the residuals themselves are lower. The K-value, however, is about 10 times larger than that for other animals in the same

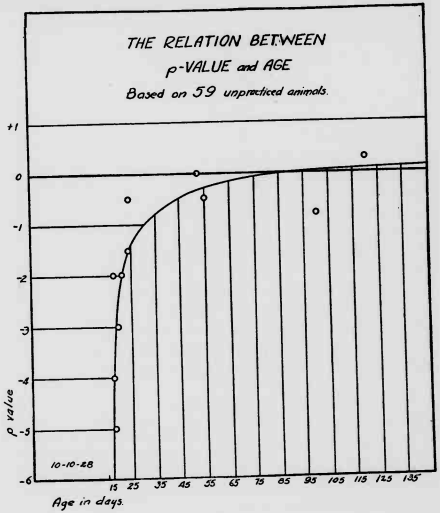

FIG. 4

problem and, more seriously, $t_1 = -35.0$ seconds, a condition theoretically formulatable but actually non-existent.

In general, the arc cotangent function is much more flexible than the hyperbola. One can ascribe various values to t_1 and leave the form of the curve unchanged. If one changes t_1 in the

hyperbola, it is necessary to make corresponding changes in a. The hyperbola in the form $t_i \frac{n_1 + p + a}{n_1 + p}$ really contains only two parameters as against 3 independent ones in the arc cotangent function. The reader can clarify this point by substituting values in (1) and (2) and plotting the corresponding curves.

Although not apparent in the fashion that the data is presented here, the graphs for the remainder (as (1) and (2)) show that for the younger rats neither function approximates the first part of the data. Since we have considerably more confidence in the terminal data, these points are given more attention in the process of fitting. When this is done with the functions for the younger rats, it appears that p becomes more and more negative as age decreases. While this is a perfectly logical situation for the arc cotangent function, for the hyperbola the calculated points cannot be determined for the early part of the experiment. The apparent trend of the p-value against age for the arc cotangent function is indicated in figure 4. The same regularity is not apparent in computations based on the hyperbola. With respect to figure 4 the exact values of the p's are questionable since the function does not fit the data of the young rats very well. In all probability, further research will show that two or more combined functions are necessary for these data.

CONCLUSIONS

1. As far as "goodness of fit" is concerned, it is immaterial whether one uses the hyperbola or the arc cotangent function for data which do not in themselves exhibit an inflection point. In these cases the hyperbolae and arc cotangent functions are approximations of each other and for proper values of the parameters will approximate each other as close or closer than either approximates the raw data.

2. For older animals—above 55 days—both functions approximate the data throughout the range, but for younger ones, the arc cotangent function gives values to low at the beginning of the problem and the hyperbola gives indeterminate values.

3. If one chooses the hyperbola which gives a good approximation to the data, the limiting value (t_l) is always lower than it should be. The arc cotangent function, on the other hand, gives values of t_l in better accord with both logical conclusions and experimental findings.

4. Negative values of p, perfectly logical in the arc cotangent function, become uninterpretable in the hyperbola.

5. The arc cotangent function, particularly with the addition of the S-parameter (Valentine, 1930) is much more flexible than the hyperbola and is therefore more valuable in fitting learning data.

6. The experimental evidence seems to indicate that the hyperbola is too steep towards the left for learning data and that the arc cotangent function is too flat. The real condition probably lies between the two, the data passing to infinity to the left, but more gradually than the hyperbola. Thus, some non-symmetrical curve would more closely approximate the actual conditions than the symmetrical arc cotangent function.

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BIOGRAPHY

Willard Lee Valentine was born December 2, 1904 at Chillicothe, Ohio. All his primary and secondary education was received in the public schools of that city. He was graduated from the Chillicothe High School in 1921. His undergraduate work was done at Ohio Wesleyan University, from which institution he received the degree of Bachelor of Arts in 1925 and Master of Arts in 1926. He was graduate assistant in psychology and mathematics during the year 1925-26 and held an instructorship in psychology there from 1926 to 1928. During 1928-29 he has been an instructor in psychology at Ohio State University and has been registered in the graduate school of that institution since 1926. During 1929 the requirements for the degree of Doctor of Philosophy have been completed.

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